ESTIMATION OF THE SPECIFIC REAL PHASE AND GROUP REFRACTIVE INDEXES BY THE ALTITUDE IN THE EARTH'S IONIZED REGION USING THE FIRST ORDER APPLETON-HARTREE EQUATIONS

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¹Abstract: The specific phase and group refractive indexes concerning the specific phase and group velocities of single and packet electromagnetic waves contain all interactions between the electromagnetic waves and the propagating medium. The determination of the specific refractive indexes vs. altitude is also a challenging and complicated problem. Based on the first-order Appleton-Hartree equations and the values of free electron density by altitude, this paper outlined the numerical estimated results of the specific real phase, group refractive indexes vs. the altitude from 100 km up to 1000 km in the ionized region. The specific real phase refractive index has a value smaller than 1, corresponding to this value, the specific phase velocity is larger than the light speed (*c*) meanwhile the value of the specific real group refractive index is larger than 1, the specific group velocity will always be smaller than light speed (*c*). These estimated results are agreed with the theory and forecasted model predicted. These results could be applied for both the experiment and theoretical researches, especially for application in finding the numerical solution of mathematics problems of Wireless Information and Wireless Power Transmissions.

Keywords: Specific real phase and group refractive indexes by altitude, The First order Appleton-Hartree equations, the Earth's ionized region, Microwave propagation.

I. INTRODUCTION

The developments of the theoretical aspects *of* the refractive indexes concerning the electromagnetic waves (EMW) propagation in the Earth's ionized region always have been studying up today. The refractive index of the EMW is an essential concept that reflects the interactions between the EMW and a given medium. Depending on the features of a given propagating medium and the forms of EMWs, the refractive index is changed and it has been discussed and formulated in different forms, such as by Sellmeyer formula and Lorentz formula [2-5]. During the time from 1927 to 1932, the essential formula for the refractive index of the Earth's atmosphere's ionized region in a magnetic field has been developed and called by the name of the Appleton-Hartree equation. This equation describes generally the refractive index for EMW propagation in a cold magnetized plasma region - the ionosphere region. Since then there were many aspects concerning this refractive index expression that have been studied and published in Literature, for example: the determination of constants being in the Appleton Hartree equation [4, 5]; the study of effect of electron collisions on the formulas by magneto-ionic theory; the development of theory, mathematical formulas concerning the complex refractive indices of an ionized medium [4, 5 and 7]; the conditions and the validity of some

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approximations related to the refractive index have also been studied including the high order ionosphere effects on the global positioning system observables and means of modeling [6, 7, 8 and 9]; the proposed model and predicted values of the refractive index in the different layers of the earth atmosphere medium [10]; the scattering mechanisms of EMW [11]; the variation of the ionosphere conductivity with different solar and geomagnetic conditions [12]; the ionosphere absorption in vertical propagation [13]; the atmospheric influences on microwaves propagation[14]; the stochastic perception of refractive index variability of ionosphere [16]; and a lot of other aspects have been studied in references [15-19].

 Recently there are also many works continuing to study deeply different problems such as determination of the specific phase and group refractive indexes in different propagating environments, the calculation of the discrete refractive indexes based on some conditions, the calculation of the refractive index at F region altitudes based on the global network of Super Dual Auroral Radar Network (SuperDARN) [17-21]. In addition, presently many attempts are devoted to researches of

 The Wireless Power Transmission (WPT) problems using high power microwaves and Laser power beams. During propagation of high power beams, the Earth atmosphere region will be ionized, this fact has generated some research problems concerning the propagating theory development of EMWs power beams with Gaussian energy distributions, the real interactions of High power beams and the Earth atmosphere this fact brought about the modified concepts of the relative permittivity, EMWs velocities, and refractive indexes [25-32, 39, 40].

So far, it has a few systematic data of the specific phase and group refractive indexes vs. altitude of the ionized region published in the Literature [10, 27, 28, 37, 42, 43]. In our previous published work [28, 39], we have studied and outlined the relative permittivity and the numerical data of the complex phase refractive index by altitude based on the free electron density (*Ne*) distribution [38]. In this paper using the first-order Appleton-Hartree equations bypassing the imaginary parts due to their values are very small, we estimated and outlined the systematic numerical results of both kinds of the real phase and group refractive indexes $(n_{ph}$ and n_{gr}) vs. the altitude concerning the single and packet EMWs forms propagating in the ionized regions from 100 km to 1000 km depending on the frequency range of from 8 MHz to 5.8 GHz.

II. THE EXPRESSIONS OF RELATIVE PERMITTIVITY AND REAL REFRACTIVE INDEXES EXPRESSIONS FOR THE EARTH'S IONIZED REGION

II.1. Briefly on electromagnetic waves propagation in the ionized region

The features of the ionosphere region strongly influence microwaves propagation. The mechanism of refraction mainly occurs in the following ways: when the EMW comes to the ionosphere region, the electric field of EMW

forces the free electrons being in the ionosphere into oscillation with the same frequency as that of the EMW. Some of the radio-frequency energy is transferred to this resonant oscillation, and the oscillating electrons will then either be lost due to recombination or will re-radiate the original wave energy. The total refraction can occur when the collision frequency of the ionosphere is less than the EMW frequency, and the electron density in the ionosphere is high enough [9, 14, 15, 25].

When the EMW frequency increases to higher values, the number of reflection decreases and then not the refraction. So there will be a defined limiting frequency (*so-called, critical frequency or plasma frequency*) where the signals could pass through the ionosphere layer [9,14, 33]. If the propagating EMW's frequency is higher than the plasma frequency of the ionosphere, then the free electrons cannot respond fast enough, and they are not able to reradiate the signal. The expression determining the critical frequency has the form: $f_{critical} = 9 \sqrt{N_e}$. Herein, N_e [m⁻³] is a free electron density being in the ionosphere region. If we do not take into account the number collision of ionized particles (O, N, H...), then the effective permittivity $(\varepsilon_{\text{eff}})$ as a function of *critical frequency or plasma frequency* (ω_p) and EMW's frequency (ω) that can be written as the following form [32, 38, 39]:

$$
\varepsilon_{\text{eff}} = \varepsilon_0 \left(I - \frac{\omega_p^2}{\omega^2} \right) \text{(a)}; \qquad \omega_p = \sqrt{\frac{N_e e^2}{m \varepsilon_0}} \quad \text{(b)} \tag{1}
$$

Based on this formula, the plasma frequency (ω_p) of the ionized region has been calculated, and its value is about 8 MHz [32-34].

II.2. The expressions of the complex relative permittivity in the ionized region

In the ionized region, the dielectric permittivity has been accepted as a complex number. Various processes are labeled on the imaginary part: ionic and dipolar relaxation, atomic and electronic resonances at higher energies. As the response of the ionized region to external fields that strongly depends on the EMW [frequency,](https://en.wikipedia.org/wiki/Frequency) the response must always arise gradually after the applied field, which can be represented by a phase difference leading to the formation of the imaginary part. The complex relative permittivity in the ionized region can be expressed in the following form [36, 38, 39]:

$$
\varepsilon_r(\omega) = I - 4\pi \frac{N_e e^2}{\varepsilon_o m_e} \frac{I}{(\omega^2 + S^2)} - i \frac{4\pi\sigma}{\omega} = \varepsilon_r'(\omega) + i\varepsilon_r''(\omega)
$$

(2)

$$
\sigma = \frac{N_e e^2}{m_e} \frac{S}{(\omega^2 + S^2)}
$$

Herein, N_e is the free electron density, ω is the angular frequency, m_e is electron mass, ε_o is the vacuum dielectric constant, σ is the conductivity, and *S* is the collision angular frequency of ionized particles in the ionized region. The $\varepsilon'_r(\omega)$ and $\varepsilon''_r(\omega)$ are denoted as the real part and imaginary part of the relative permittivity, respectively. Based on the graphic curves of free electron density by altitude in the ionized region, the different kinds of conductivities and relative permittivity vs. the altitude

have been estimated [38, 39].

II.3. The first-order expressions of Appleton-Hartree formulas for calculation of the specific real phase and group refractive indexes

As known, the refractive index offered in the Literature is often undertaken as a general refractive index determined by $n = c/v$. Herein, *c* is the speed of light, *v* is the related velocity of EMW. *This concept is not* often distinguished clearly from the specific phase. group, energy velocities concerning the different forms of the single wave, packet waves, power beams EMWs propagating in given medium [22-24, 28]. This concept is only valid and used for the ideal medium (linear medium) corresponding to an ideal vacuum (homogeneous, isotropic, linear) where all forms of EMWs travel with the same velocity [1, 2, 3, 14, 36, 37]. In fact, for the reality medium, depending on the different types of the EMWs (single sinusoidal wave, packets wave, and distributed waves power beams…), the EMWs will travel with different velocities (the phase velocity, group velocity, particle velocity, and energy velocity) [22-24]. Here it is *so-called the specific velocity* corresponding to the related refractive index, it is *so-called the specific refractive index.* The specific refractive index expression is given by $n_x = c/v_x$ where n_x is denoted by the specific phase, group, or energy refractive index that is concerning the specific velocity (v_x) of phase velocity for single EMW, group velocity for packet EMWs, or energy velocity for energy power beam, respectively. The general original equation of the complex refractive index for ionosphere region, so-called the Appleton-Hartree Equation based on the work of Budden (1985) is written as the following form [33]:

33]:
\n
$$
n^{2} = 1 - \frac{A}{1 - jC \left(\frac{B_{T}^{2}}{2(1 - A - jC)}\right)^{\frac{1}{2}} \left(\frac{B_{T}^{4}}{4(1 - A - jC)^{2}} + B_{L}^{2}\right)^{\frac{1}{2}}}
$$
\n(4)

Herein the dimensionless quantities A, B, and C are defined as follows:

$$
A = \frac{f_N^2}{f^4} , B = \frac{f_B}{f} , B_L = \frac{f_B}{f} \cos \theta ,
$$

$$
B_r = \frac{f_B}{f} \omega_B \sin \theta \; , \quad C = f_c / f \text{ where } f_N \text{ is the}
$$

angular plasma frequency: $f = \left(\frac{N_e e^2}{\lambda_e}\right)^{\frac{1}{2}}$ $\overline{0}$ $E_N = \left(\frac{N_e.e^2}{\varepsilon_0 m_e}\right)$ $f_N = \left(\frac{N_e.e}{\varepsilon_0 m_e}\right)$ $=\left(\frac{N_e.e^2}{\varepsilon_0 m_e}\right)^{\frac{1}{2}};$

 f_B is the electron gyro-frequency: $f_B = \frac{B_o}{m_e}$ $f_B = \frac{B_o \cdot e}{m_e}$ (f) is the

frequency of the EMW, *θ* is the angle between the propagation direction and the geomagnetic field, *N^e* is the *free* electron density in the ionosphere region due to particles (O, N, H…) ionized, *B* is the magnitude of the magnetic field vector, the meaning of other symbols have mentioned in above. When *f* comes to a remarkably high value (>100 MHz) or infinite, the terms of imaginary in the Appleton-Hartree Equation (4) will be neglected. Besides, if the collision effects of the particles are not taken into consideration, after yielding Eq. (4), the expressions of the real specific phase and group refractive indexes *(nph* and n_{gr} can be derived, they have the following forms [8, 9]:

$$
n_{ph} = I - \frac{f_p^2}{2f^2} \pm \frac{f_p^2 f_g \cos \theta}{2f^3} - \frac{f_p^2}{4f^4} \left[\frac{f_p^2}{2} + f_g^2 (I + \cos^2 \theta) \right]
$$
(5)
\n
$$
n_{gr} = I + \frac{f_p^2}{2f^2} \mp \frac{f_p^2 f_g \cos \theta}{2f^3} + \frac{3f_p^2}{4f^4} \left[\frac{f_p^2}{2} + f_g^2 (I + \cos^2 \theta) \right]
$$
(6)
\n
$$
f_p^2 = \frac{N_e e^2}{4\pi^2 \varepsilon_0 m_e}
$$
(a) ; $f_g = \frac{eB}{2\pi m_e}$ (b) (7)

Herein, n_{ph} and n_{gr} are the specific real phase for single EMW and group refractive indexes for packet EMWs, respectively. The Eqs. (5), (6) are so-called, the specific high-order real phase and group refractive indexes of the Appleton-Hartree formulas. We observed that Eqs. (5) and (6) have opposite signs before the three terms after the first term with the value of 1. The waves with the upper signs after the second term in Eqs. (5) and (6) are called the ordinary waves (*O-wave*) and are left-hand circularly polarized waves. In contrast, the waves with the lower signs are called the extraordinary waves (*X-wave*) and are righthand circularly polarized [1, 6, 8, 9, 28, 37, 44]. If we take only the effects of the free electron density (N_e) in the ionosphere region into consideration, the equations of the high order refractive indexes of Appleton-Hartree formulas in Eqs. (5), (6) will become to simple forms, which are named *the first-order expressions of the specific real phase and group refractive indexes* [18, 37]. After

substituting the constants symbols of *e, m_e,* π *, and* ε _{*o*} into Eqs. (5), (6), these expressions will be reduced to the following approximated forms [6,8,9,37]:

$$
n_{ph} \approx I - \frac{J_p^2}{2f^2} = I - \frac{Nee^2}{8\pi^2 \epsilon_0 m_e f^2} = I - 40.31 \frac{N_e}{f^2}
$$
(8)

$$
n_{gr} \approx I - \frac{J_p^2}{2f^2} = I - \frac{Nee^2}{8\pi^2 \epsilon_0 m_e f^2} = I + 40.31 \frac{N_e}{f^2}
$$
(9)

The values of n_{ph} and n_{gr} by altitude can be determined based on the N_e values vs. altitude at different given frequencies.

III. RESULTS AND DISCUSSIONS

III.1. The variation of the relative permittivity vs. altitude

Based on Eqs. (2) & (3) and the outlined graphic free electron density (*Ne*) distribution by altitude in the ionosphere region [38], the relative permittivity concerning the two kinds of the Pedersen conductivity (σ_p) and Field– Aligned conductivity (σ_{FA}) has been estimated and outlined in the tables [39]. These results are redrawn on Figs.1&2 for a more clear review to setting up our proposal estimating condition for this paper.

Figure 1. The numerical results of relative complex permittivity vs. altitude from 100 km to 1000 km based on Field-Aligned conductivity (σF.A), the real part data (a), and the imaginary part data (b).

Figure 2. The numerical results of relative complex permittivity vs. altitude from 100 km to 1000 km based on Pedersen conductivity (σ_p) *, the real part data (a), and the imaginary part data (b).*

From Figs.1&2 we see clearly that the imaginary parts of complex permittivity's values are very small in the ranges of 10^{-7} for the Field-Aligned conductivity (σ_{FA}) case and 10^{-12} for Pedersen conductivity (σ_p) case. This

fact supports our proposal estimating conditions: We can ignore the imaginary parts in Eq. (4) as well as the higherorder terms being in Eqs.5&6 for numerical estimation of the real phase and group refractive indexes in this work.

III.2. The estimated results of the specific real phase and group refractive indexes vs. altitude in the ionized region from 100 km to 1000 km

Using Eqs. (8) and (9) with the same numerical calculated method with replacing the values of the free electron density by altitude that outlined in the tables in the work [39] we will have estimated the systematic numerical results of both the specific real phase and group refractive indexes vs. altitude from 100 km to 1000 km concerning the specific velocities of the single EMW and packet EMWs. The obtained results are outlined in Figs. $3 \& 4$ for four different frequencies of 8 MHz, 100 MHz, 2.45 GHz, and 5.8 GHz.

Figure 3. The specific real phase (nph) and real group (ngr) refractive indexes vs. altitude from 100 km to 1000 km at the EMW frequencies 8 MHz (a) and 100 MHz (b).

Figure 4. The specific real phase and group refractive indexes vs. altitude from 100 km to 1000 km at the EMW frequencies 2,45GHz (a) and 5.8GHz (b).

Figure 5. The estimated results of the real phase and group refractive indexes in comparison with two frequencies at 2.45 GHz and 5.8 GHz

From the obtained results we observed that the values of the specific real phase refractive index varied strongly along with the altitude in the range of from 150 km to 500 km. At 250 km altitude, their values are smaller than 1. For example, its value is 0.2 at 8 MHz, and increased to

0.9999985 at 5.8 GHz; corresponding to these values, *the specific phase velocities concerning the propagation of a single EMW form in this region could be larger than the light speed (c). This result is opposite the Einstein principle, but indeed at some special propagating environments, the phase velocity could be larger than the light speed. This fact could be accepted for the phase velocity of single EMW propagation when it is not contained information as the approach* predicted [22, 23].

The obtained results in Figs.3, 4, 5 also show the values of the specific real group refractive indexes concerning the propagation of the packet EMWs in the ionized region that are larger than 1, for example, at the altitude of 250 km, its value is 1.8 for 8 MHz frequency EMW, it varied to the value of 1.0000015 at 5.8 GHz frequency EMW; corresponding to these values, the specific group velocities in this region will always be smaller than light speed (*c*) due to the propagation of packet EMWs is usually contained energy/information [22, 23]. *In practice, depending on the given form of EMW, the EMW could propagate with its own specific phase, group, or energy velocity; this will determine the value of the own specific phase, group, or energy refractive index, respectively. Indeed it is hard to distinguish or point out clearly which kind of the EMW's specific velocity is really propagated*. Therefore the related refractive index so far is often labeled by the general refractive index, not by a defined specific refractive index. This situation together with the result of the specific real phase velocity has a value larger than light speed (*c*) these facts should be studied and explained more clearly in next time.

Our obtained results of specific refractive indexes here are in orders similar to the values of refractive indexes predicted model and discrete values determined at different altitude and local positions published in Literature. Our results are listed in comparison with several published results of refractive indexes computed or measured at different regions and conditions, as in Table 1 in bellow [10, 16, 21, 42, 43, 45].

IV. CONCLUSIONS

 - We have outlined briefly some research - development activities more in detail concerning the specific phase and group refractive indexes given by the relation $n_x = c/v_x$ concerning the specific phase and group velocities of the single EMW and packet EMWs forms in the Earth atmosphere's ionized region, respectively.

- The systematic numerical estimation of the real refractive indexes by altitude from 100 km to 1000 km in the ionized region is firstly carried out at four frequencies of 8 MHz, 100 MHz, 2.45 GHz, and 5.8 GHz based on the first-order Appleton-Hartree equations and based on the data of the free electron density (*Ne*) vs. altitude. These estimated results are agreed with the theory and forecasted model published in Literature.

- The specific real phase refractive index in the ionized region has a value smaller than 1, corresponding to the specific phase velocity could be larger than the light speed (*c*), this result could be accepted for phase velocity of the propagating single EMW not containing the information as the theory predicted. Meanwhile, the value of the specific real group refractive index is larger than 1, corresponding to the specific group velocity will always be smaller than light speed (*c*). This is explained by the propagation of packet EMWs always containing information/energy, as predicted by theory.

-The obtained data in this paper have significant

meanings: this gives a general view about the variations of the specific refractive indexes in whole the ionized region. These estimated data could be used for the discussion as well as used to replace into Maxwell equations for investigation and/or calculation of the numerical solution of the mathematical problems of WIT, GPS, and WPT to determine the transfer efficiencies. These problems should be continuously discussed and studied more detail in the next time.

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TÍNH TOÁN PHẦN THỰC HỆ SỐ CHIẾT SUẤT PHA VÀ NHÓM ĐẶC THÙ DỌC THEO CHIỀU CAO LỚP KHÍ QUYỂN ION HÓA SỬ DỤNG CÁC PHƯƠNG TRÌNH BẬC NHẤT APPLETON-HARTREE

Tóm tắt: Các hệ số chiết suất pha và nhóm liên quan đến tốc độ pha và tốc độ nhóm đối với sóng điện từ đơn và bó sóng điên từ truyền trong lớp khí quyển ion hóa chúng chứa đựng tất cả các tương tác giữa sóng điện từ và môi trường truyền sóng. Việc xác định đúng giá trị các hệ số chiết suất đặc thù dọc theo chiều cao lớp ion hóa của tầng khí quyển Trái đất luôn luôn được quan tâm, tuy nhiên đó cũng là vấn đề thách thức lớn và rất phức tạp vì các thông số của lớp ion hóa luôn thay đổi ngẫu nhiên phụ thuộc rất nhiều vào vị trí trí, mùa, thời tiết, bức xạ mặt trời và thời gian vào ban ngày và ban đêm. Đến nay đã có một số mô hình dự đoán và nhiều công trình xác định hệ số chiết suất rời rạc trên các khía cạnh lý thuyết và thực nghiệm khác nhau , tuy nhiên đánh giá -tính số một cách hệ thông hệ số chiết suất pha và nhóm theo theo chiều cao thì chưa thấy có nhiều kết quả công bố. Trên cơ sở các phương trình Appleton-Hartree về hệ số chiết suất giản ước bậc nhất với giả thiết bỏ qua các thành phần số phức là rất nhỏ, bài báo này lần đầu tiên cố gắng xác định và trình bày một cách có hệ thống kết quả tính số phần thực các hệ số chiết suất pha đối với sóng đơn và hệ số chiết suất nhóm đối với bó sóng điện từ phụ thuộc vào tần số vi ba và chiều cao lớp ion hóa khí quyển trái đất từ 100 km đến 1000 km. Những kết quả này cho bức tranh tổng quát về sự biến đổi của hệ số chiết suất phụ thuộc vào tần số và chiều cao, và trên cơ sở này có thể phát triển để ứng dụng trong các nghiên cứu thực nghiệm và lý thuyết, đặc biệt có thể ứng dụng trong giải bài toán truyền sóng tìm lời giải số về truyền thông tin không dây và truyền chùm tia công suất cáo không dây giữa hai điểm trong bầu khí quyển trái đất .

Từ khóa: phần thực hệ số chiết suất pha và nhóm theo chiều cao; phương trình Appleton-Hartree bậc nhất, Miền ion hóa bầu khí quyển trái đất, truyền sóng điện từ.

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