

INCREASING THE ACCURACY OF NONLINEAR CHANNEL EQUALIZERS USING MULTIKERNEL METHOD

Viet-Minh Nguyen

Posts and Telecommunications Institute of Technology

Abstract: In previous articles, we proposed single kernel and multikernel equalizers for nonlinear satellite channels with significant improvements in performance. The results demonstrated that the advantages of kernel equalizers over radius basis function neural equalizers are the ability to achieve overall convergence, which results in smaller output errors. However, the limitation of single kernel equalizers is that the output errors are still quite large. Multikernel equalizers can overcome this disadvantage but the calculation is quite complex. To simplify the computation, this paper proposes a multikernel equalizer based on Online Multi-Kernel Normal LMS, MKNLMS, algorithm.

Keywords: kernel method, kernel adaptive filters, multikernel equalizers.

I. INTRODUCTION

Nowadays, the Orthogonal Frequency-Division Multiplexing (OFDM) satellite information systems are considered to be strong nonlinear systems. Under the influence of radio transmission medium, the nonlinearity of the channel causes the signal to be intercepted between the symbols, (InterSymbol Interference – ISI), and the interference between the subcarriers, (InterCarrier Interference – ICI). Signal predistortion techniques at the transmitters [11] or equalizers at the receivers can be used to eliminate these interferences. The proposed control algorithms usually use the Volterra series. These algorithms are represented in high order series [8] therefore they are extremely complex. Over the past ten years, adaptive nonlinear equalizers are being used in satellite channels [8]. These equalizers mainly use artificial neural networks [8] [11] and Radial Base Function - RBF networks are the most commonly used method. RBF equalizers, with simple structures, have the advantage of being adequate for nonlinear channels.

However, their most basic disadvantage is that only the optimal local root can be found. Therefore, the output errors will be very large when these equalizers are used in OFDM satellite information systems. To overcome this disadvantage, kernel equalizers have been proposed with the application of kernel method to traditional equalization algorithms for the purpose of simplifying computation and thus improving the equalization efficiency [6] [7] [9] [10].

In this paper, we propose a new equalization method using multikernel technique which operates based on adaptive KLMS (Kernel Least Mean Squares) algorithm. Because this method uses the gradient principle therefore the computation is simple and effective [11]. This equalization algorithm is mainly based on LMS algorithm and kernel standardized with accepting consistent criteria for directory design [12].

Basically, the LMS multikernel algorithm is still based on gradient principle. However, due to the specificity of the multikernel, there are different application hypotheses. In [1], to restrain imposing optimal weight, the authors used a port fuction softmax $\psi_k(n)$, therefore limits the application areas of the equalizer. In [2], the authors developed a multikernel learning algorithm based on the results of Bach et al. 2004 [3] and the extension of Zien and Ong 2007 [13]. The optimization tool is based on Shalev-Shwarts and Singer 2007 [14]. This is a generic framework for designing and analyzing the most statistic gradient descent algorithm. However, they are not commonly used for the fuctions with strong convexity. Do et al. 2009 [15] proposed the Pegasos algorithm, which has relatively good convergence with small λ . The disadvantage of this algorithm is that it requires knowing the upper limit of the optimal root.

Corresponding author: Viet Minh Nguyen

Email: minhvn@ptit.edu.vn

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In this paper, we propose an algorithm for multikernel equalizers based on LMS algorithm that does not require the above factors to make the computation more simple, while the convergence rate will be adjusted based on the algorithm's control step size. The LMS multikernel algorithm makes the output error of the equalizer smaller than the single-kernel equalization, therefore it is consistent with the equalizers in OFDM satellite systems.

The structure of this paper is presented as follow: Section 2: Kernel and properties; Section 3: Multikernel equalization based on LMS algorithm; Section 4: Equalization performance evaluation and Section 5: Conclusion.

II. KERNEL AND PROPERTIES

Firstly, kernel is defined as a function k with x, z of a non-empty set X satisfying the condition as below [11]:

$$k(x, z) = \langle \phi(x), \phi(z) \rangle \quad (1)$$

Here ϕ is a mapping from set X to Hilbert space F , commonly known as the characteristic space:

$$\Phi: x \in X \rightarrow \phi(x) \in F \quad (2)$$

Some features of the kernel function:

Function $k: X \times X \rightarrow R$ is continuous or can be counted, can be expanded with scalar product in Hilbert space F :

$$k(x, z) = \langle \phi(x), \phi(z) \rangle \quad (3)$$

If and only if satisfies the positive semi-definite characteristic.

Has two functions:

$$f_{F1}(\cdot) = \sum_{i=1}^{l_1} \alpha_i k(x_i, \cdot); f_{F2}(\cdot) = \sum_{j=1}^{l_2} \beta_j k(x'_j, \cdot) \quad (4)$$

Here $l_1, l_2 \in N; \alpha_i, \beta_j \in R; x_i, x'_j \in H$ then:

$$\langle f_{F1}(\cdot), f_{F2}(\cdot) \rangle \geq \sum_{i=1}^{l_1} \sum_{j=1}^{l_2} \alpha_i \beta_j k(x_i, x'_j) \quad (5)$$

Some common kernels [11]:

The Gaussian kernel:

$$k(x, z) = \exp\left(-\frac{\|x-z\|^2}{2\sigma^2}\right) \quad (6)$$

The polynomial kernel:

$$k(x, z) = (\langle x, z \rangle + R)^d \quad (7)$$

III. MULTIKERNEL EQUALIZATION BASED ON LMS ALGORITHM

Consider a simple information system model in Figure 1, which has the effect of linear distortion represented by linear filter, the effect of nonlinear distortion represented by nonlinear filter and the additive noise. The input signal of each component is shown in Figure 1.

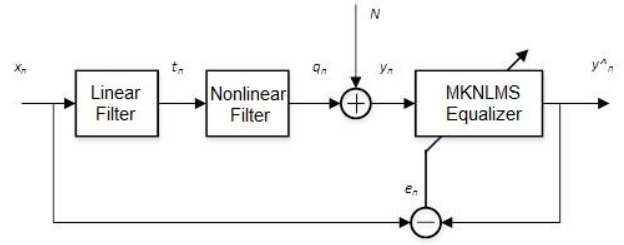


Figure 1. Information system model with KLMS equalizer

The equalization block can be separated and demonstrated as Figure 2.

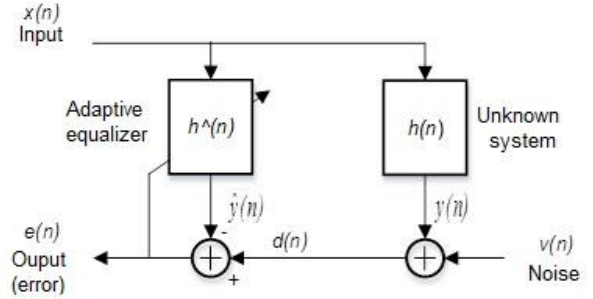


Figure 2. KLMS equalization model

Assume that we have an input-output chain:

$$\{(x_1, d_1), (x_2, d_2), \dots, (x_n, d_n), \dots\}$$

The goal of the equalizer is to minimize the output error:

$$L(w) = E[|d_n - f_w(x)|^2] \rightarrow \min \quad (8)$$

Therein $f_w(x)$ is the mapping of the equalizer with its coefficients, w :

$$f_w(x) = w^T x; w \in R^N \quad (9)$$

N is the kernel quantity of the equalizer.

$$e(n) = d(n) - w^T(n-1)x(n) \quad (10)$$

Here the paper develops an algorithm to calculate the weights of the equalizer to satisfy (8). Denote $e(n)$ is the given error at the iteration step n

Based on given training data $\{(x_i, d_i)\}$, $L(w)$ and the most decent method, we have:

$$-\nabla L(w) = 2E[(d(n) - w^T(n-1)) \cdot x(n)] = 2E[e(n)x(n)] \quad (11)$$

Approximate the value $E[e(n)x(n)] \approx e(n)x(n)$

This leads to the equation for updating the weights of the equalizer in the most decent direction:

$$w(n) = w(n-1) + \mu e(n)x(n) \quad (12)$$

Therein μ indicates the control step size of the algorithm. The algorithm is expressed as follow:

Begin: $w(0) = 0$

Step 1: given (x_1, d_1)

$$2: f(x(1)) = w^T(0)x_1 = 0$$

$$3: e_1 = d(1) - f(x(1)) = d_1$$

$$4: w(1) = w(0) + \mu e(1)x(1) = \mu e(1).x(1)$$

5: given (x_2, d_2)

Perform as step 2 to step 4; achive $w(2)$

In (12) choose the value μ satisfy the below condition:

$$0 < \mu < \frac{1}{\lambda_{max}} \quad (13)$$

To ensure that (12) always converge with probability equal to 1. Here λ_{max} is the maximum eigen value of:

$$R = E\{x(n).x^T(n)\} \quad (13)$$

Consider some special cases:

1. When the magnitude of the input vector is large, the weight vector w is much varied. Therefore to solve the above problem we have to standardize this vector. The normalized LMS algorithm is constructed in the sense that the optimal problem is constrained as follows:

The input vector $x(n)$, desired response $d(n)$ and the filter weight $w(n)$ are given. Find the weight vector of the equalizer $w(n+1)$ to minimize the Euclidean square of the difference $w(n+1) - w(n)$. This problem is solved by using Lagrange multiplier to give us the update equation [4]:

$$w(n+1) = w(n) + \frac{2\mu e(n)}{\|x(n)\|^2} x(n) \quad (14)$$

This equation will converge with $0 < \mu < 1$

2. Case: when $\|x(n)\|$ is small

In this case, it will be difficult to compute (14) and it usually requires numerical method. A highly practical update method is used to overcome this problem [4] [5]:

$$w(n+1) = w(n) + \frac{2\mu e(n)}{\alpha + \|x(n)\|^2} x(n) \quad (15)$$

Here $\alpha > 0$

Calculating based on the kernels:

$$\text{Knowing that: } f(x(n+1)) = w^T(n)x(n+1)$$

With $w(0) = 0$ we have:

$$f(x(n+1)) = w^T(n)x(n+1) = \mu \sum_{i=1}^n e(i)x^T(i)x(n+1) \quad (16)$$

Here

$$e(n) = d(n) - \mu \sum_{i=1}^n e(i)x^T(i)x(n) \quad (17)$$

When using the kernels we have new sample array:

$$\{(\phi(x(1), d(1))), \dots, (\phi(x(n), d(n)))\}$$

Function $f(x(n))$:

$$f(x(n)) = \langle w, \phi(x(n)) \rangle; w \in H \quad (18)$$

The target function:

$$L(w) = E \left[|d(n) - f(x(n))|^2 \right] = E \left[|d(n) - \langle w, \phi(x(n)) \rangle|^2 \right] \quad (19)$$

Here we set:

$$e(n) = d(n) - f(x(n)) \quad (20)$$

$$\nabla L(w) = -2E[e(n)\phi(x(n))] \quad (21)$$

Approximate:

$$\nabla L(w) = -2e(n)\phi(x(n)) \quad (22)$$

Hence we have the weighting algorithm of the equalizer based on the kernels:

$$w(n) = w(n-1) + \mu e(n)\phi(x(n)) \quad (23)$$

Algorithm

Begin: $w(0) = 0$

Step 1: $w(1) = \mu e(1)\phi(x(1))$

$$w(2) = \mu e(1)\phi(x(1)) + \mu e(2)\phi(x(2))$$

...

$$w(n) = \mu \sum_{i=1}^n e(i)\phi(x(i))$$

At each instance time n we have:

$$\begin{aligned} f(x(n)) &= \langle w(n-1), \phi(x(n)) \rangle \\ &= \mu \sum_{i=1}^{n-1} e(i) \langle \phi(x(i)), \phi(x(n)) \rangle \\ &= \langle \mu \sum_{i=1}^{n-1} e(i) k(x_i, x_n) \rangle \end{aligned} \quad (24)$$

With the NLMS normalization algorithm, we have

$$w(n) = w(n-1) + \frac{\mu e(i)}{k(x(i), x(i))} \phi(x(i)) \quad (25)$$

We then develop a sparsification multikernel NLMS algorithm based on a consistent basis as follow:

The MKNLMS algorithm

Input: Data (x_n, y_n) and number N

Output: Expression $w = \sum_{k=1}^N \alpha_k k(\cdot, u_k)$, with $\alpha_k = \mu e_k$

Begin: $f^0 = 0$, n : learning step, μ : Parameter of learning step

Define: vector $\alpha = 0$, matrix $D = \{ \}$ and the parameters of kernel function

for $n = 1 \dots N$ **do**

if $n == 1$ **then**

$$f_n = 0$$

else

Calculating the equalizer output: $f_n = \sum_{k=1}^M \alpha_k k(u_k, x_n)$
end if
 Calculating the error: $e_n = d_n - f_n$
 $\alpha_n = \frac{\mu e_n}{k(x(n), x(n))}$
 Check the sparsification condition
if the sparsification condition is satisfied then
 $M = M + 1$
 Write a new center $u_M = x_n$ in the center list
 $D = \{D, u_M\}, \alpha^T = \{\alpha^T, \alpha_n\}$
end if
end for

IV. EQUALIZATION PERFORMANCE EVALUATION

This section will show the performance of the proposed multikernel equalization solution based on the MKNLMS algorithm. The algorithm uses two Gaussian kernel ($M = 2$) with parameters ξ_1, ξ_2 . MSE is calculated based on an arithmetic mean of 500 executions. To see the effectiveness of the solution, we compare the results to traditional NLMS single kernel and traditional LMS solutions.

The equalization is performed for the dynamic channel described by the sudden channel change in the 500th sample. The transmitter sends binary symbols $(s_n)_{n \in N} \subset \{\pm 1\}$ with equal probabilities, the received signal with $n \leq 500$ is created from $y_n = x_n - 0,9x_n^2 + v_n$ with $x_n = s_n + 0,5s_{n-1}$ [11], and with $n > 500$ it will be created from $y_n = x_n + x_n^2 + v_n$ with $x_n = s_n - 0,2s_{n-1}$. The channel is affected by AWGN noise $v_n \in \mathbb{R}$ with $SNR = 10 \log_{10}(E\{z_n^2\}/E\{v_n^2\}) \geq 20dB$ with $z_n = x_n - 0,9x_n^2$. The noise power is considered constant as the power of the received signal increases due to channel change.

The equalizer problem is to restore the transmitted symbol $(s_n)_{n \in N}$ from the received symbol $(y_n)_{n \in N}$. In the information system, owing to the transmitted pilot symbols, we always have $(s_n)_{n \in N}$ to adapt to the nonlinear equalizer. We set $u_n = [y_{n+D}, y_{n+D-1}, \dots, y_{n+D-L+1}]^T$ with $L = 5$ and $d_n = s_n$.

We compare the performances of the proposed MKNLMS algorithm with the KNLMS and linear LMS algorithms. The parameter set used in computation is given in Table 1. The average directory size is $\bar{r} = 34,8$ for the algorithms.

Table 1. Setting the parameters for the equalizers to evaluate their performances

LMS	Step size: $1,0 \times 10^{-2}$
KNLMS (1)	$\eta = 0,2; \rho = 3,0 \times 10^{-2}; \xi = 0,2$
KNLMS (2)	$\eta = 0,2; \rho = 3,0 \times 10^{-2}; \xi = 0,5$
MKNLMS	$\eta = 0,2; \rho = 6,0 \times 10^{-2}; \xi_1 = 0,2; \xi_2 = 2; \delta = 0,36$

Figure 3 shows the results of the computation.

It is clear to observe that the MKNLMS has domination MSE performance over KNLMS (I) in case of static channel. Tracking the performance of the KNLMS (II) after the channel changed, it can be seen that the use of slightly different kernel parameters instead of the optimal parameter causing severe performance degradation. The performance is even worse than the LMS linear adaptive equalizer. With changing channel, the MKNLMS exhibits good adaptability and quickly attains the lowest stable MSE, approximately 10^{-1} , after about 5000 iterations

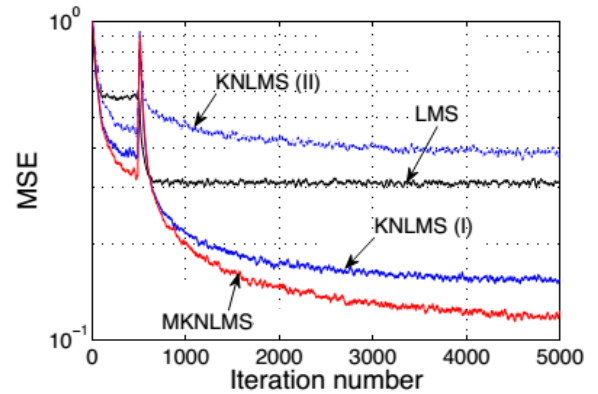


Figure 3. MSE performance comparison between the equalizers

V. CONCLUSION

The kernel equalization method is a good solution for the changing nonlinear channel equalizers. To improve the kernel equalizers, this article introduced an adaptive multikernel nonlinear equalization solution based on the Online MKNLMS algorithm. The adaptive MKNLMS multikernel equalizer shows a significant improvement in MSE performance compares to nonlinear channel equalizers using single kernel and the ability to trace the changing channel is quite good. With this feature, the MKNLMS equalizer is adequate for the changing nonlinear satellite channel such as multimedia satellite channels owing to the ability to reduce interference and nonlinear distortion in these systems./

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Viet-Minh Nguyen, received the BS degree and MS degree of electronics engineering from Posts and Telecommunications Institute of Technology, PTIT, in 1999 and 2010 respectively. His research interests include mobile and satellite communication systems, transmission over nonlinear channels. Now he is PhD student of telecommunications engineering, PTIT, Vietnam.