A SIMPLE PATH TRACKING CONTROL OF TWO-WHEELED MOBILE ROBOT

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Abstract: In this paper, a nonlinear path tracking controller based on a combination of kinematics and torque backstepping methods is applied to a nonholonomic Two-Wheeled Mobile Robot (WMR). The mobile robot is considered in terms of dynamics model in Cartesian coordinates and its parameters are exactly known. To achieve the controller, the tracking errors are defined, the control inputs for the kinematics controller are designed, and the torque controller is designed to guarantee that the errors converge to zero asymptotically. The simulation results are included to illustrate the performance of the control law.

Keywords: Wheeled Mobile Robot (WMR), path tracking, backstepping.

I. INTRODUCTION

Mobile robots have been studied and are increasingly applied in practical fields: industry manufacturing, medical services, military tasks, house operations, planetary exploration, entertainment, and so forth. Robotics research is highly interdisciplinary requiring the integration of control theory with mechanics, electronics, artificial intelligence, communication and sensor technology.

For mobile robots control strategies, the navigation problem is one of the most important operations. It may be divided into three basic problems: trajectory tracking, path following and point stabilization. Much of research have been written about solving the motion problem of the above under nonholonomic constraints using kinematics model of a mobile robot; some others, about the of integration of the nonhonomic controller and the dynamics of the mobile robot; less others still have been focused on robustness and control in presence of uncertainties in the dynamical model

In literature, Fierro et al., developed a combined kinematics and torque control law using backstepping approach, and asymptotic stability is guaranteed by Lyapunov theory; particularly, a general structure for controlling a mobile robot was derived. The structure can accommodate difference control techniques, from conventional computed-torque controller to robust-adaptive controller [3]. T. Fukao et al., proposed the integration of a kinematics controller and a torque controller for the dynamic model of a nonholonomic mobile robot with unknown parameters of the wheels [6]. Jung-Min Yang et al., 1998, proposed a new sliding mode control which is robust against initial condition errors, measurement disturbances and noise in the sensor data to asymptotically stabilize to a desired trajectory by means of the computed-torque method [4]. Additionally, it is clear that the mobile robot requires accurate sensing of the environment, intelligent trajectory planning, and high precision control.

In this paper, the trajectory tracking problem for a two-wheeled mobile robot is considered. This is the first step of the development of an intelligent indoor mobile robot. A backstepping control approach is applied to design a controller in terms of dynamics model: first, feedback velocity control inputs are designed for the kinematics steering system to make the position errors asymptotically stable; then, a computed-torque controller is design such that the mobile robot’s velocities converge to the given velocity inputs. The simulation results have been done to show the effectiveness of the proposed controller.

II. MOBILE ROBOT MODELLING

In this section, the dynamics of a two-wheeled mobile robot is considered with the nonholonomic constraints in relation with its coordinates and the reference path.

A. A Nonholonomic Two-Wheeled Mobile Robot

A mobile robot system having an n-dimensional configuration space with generalized coordinates $(q_1,...,q_n)$ and subject to m constraints can be described by [3]

$$M(q)\ddot{q}+V(q,\dot{q})\dot{q}+F(\dot{q})+G(q)+\tau_d=B(q)\tau-A^T(q)\lambda \tag{1}$$

where

$M(q)\in R^{nxn}$ is a symmetric and positive definite inertia matrix, $V(q,\dot{q})\in R^{nxn}$ is the centripetal and coriolis matrix,

$F(\dot{q})\in R^{nx1}$ denote the surface friction, $G(q)\in R^{nx1}$ is the gravitational vector, τ_d denotes bounded unknown disturbance including unknown unstructured dynamics, $B(q)\in R^{nxr}$ is a input transformation matrix, $\tau\in R^{rx1}$ is the control input vector, $A(q)\in R^{mxn}$ is the matrix related with nonholonomic constraints, and $\lambda\in R^{mx1}$ is the vector of constraint forces.

We consider that all kinematics equality constraints are independent of time, and can be expressed as follows:

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Manuscript received: 5/2023, revised: 6/2023, accepted: 7/2023.

$$A(q)\dot{q} = 0 \tag{2}$$

Let $S(q)$ be a full rank matrix $(n-m)$ formed by a set of smooth and linearly independent vector fields spanning the null space of $A(q)$, that is,

$$S^T(q)A^T(q) = 0 \tag{3}$$

According to (2) and (3), it is possible to find an auxiliary velocity vector $v \in R^{n-m}$ such that

$$\dot{q} = S(q)v \tag{4}$$

The model of two-wheeled mobile robot shown in Fig. 1 is of a typical nonholonomic system. The mobile robot has two driving wheels mounted on the same axis, and one rear wheel. The posture of the mobile robot can be described by three generalized coordinates:

$$q = [x \quad y \quad \phi]^T$$

We assume that the wheels are purely rolling and do not slip, that is, the robot can only move in the direction normal to the axis of the driving wheels. Analytically, the mobile base satisfies the following condition [3]

$$\dot{y}\cos\varphi - \dot{x}\sin\varphi = 0 \tag{5}$$

The kinematics equations of motion in terms of linear velocity and angular velocity of the mobile robot are

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos\varphi & 0 \\ \sin\varphi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{6}$$

where $|v| \le v_{\max}$ and $|\omega| \le \omega_{\max}$; v_{max} and $\omega_{\max}$ are the maximum linear and angular velocities of the mobile robot.

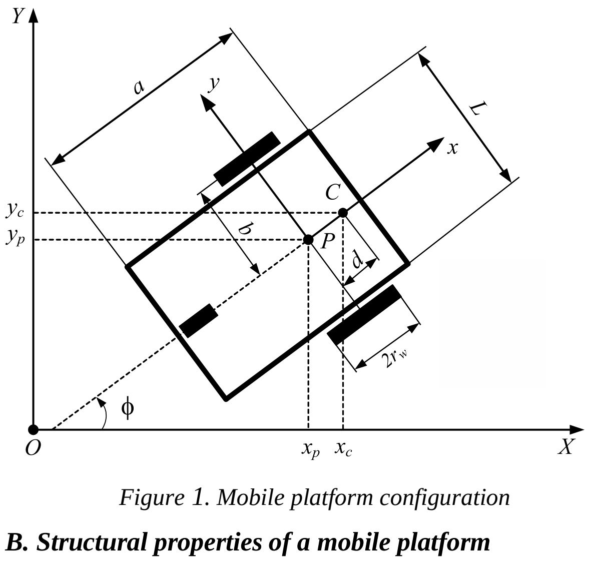

Figure 1. Mobile platform configuration

B. Structural properties of a mobile platform

The system (1) is now transformed into a more appropriate representation for control purposes. Differentiating Eq. (4), substituting the result into Eq. (1), and then multiplying by S^T, we can eliminate the constraint matrix $A^T(q)\lambda$. Therefore, the complete equations of motion of the nonholonomic mobile platform are given by

$$\dot{q} = S.v \tag{7}$$

$$S^T MS\dot{v} + S^T(M\dot{S} + V_m S)v + \bar{F} + \bar{\tau}_d = S^T B\tau \tag{8}$$

Eq. (8) can be rewritten as follows

$$\bar{M}(q)\dot{v} + \bar{V}_m(q,\dot{q})v + \bar{F}(v) + \bar{\tau}_d = \bar{B}\tau \tag{9}$$

$$\bar{\tau} \equiv \bar{B}\tau \tag{10}$$

where $\bar{M}(q) \in R^{rxr}$ is a symmetric, positive definite inertia matrix, $\bar{V}(q,\dot{q}) \in R^{rxr}$ is the centripetal and coriolis matrix, $\bar{F}(v) \in R^{rx1}$ is the surface friction, $\bar{\tau}_d$ denotes bounded unknown disturbances including unstructured unmodeled dynamics, and $\bar{\tau} \in R^{rx1}$ is the input vector. If $r = n-m$, it is easy to verify that $\bar{B}$ is a constant nonsingular matrix that depends on the distance between the driving wheels R and the radius of the wheel r.

The complete dynamics system consists of the kinematics system (7) and an extra dynamics (8). Let u be an auxiliary input, then by applying the nonlinear feedback

$$\begin{aligned} \tau &= f_\tau(q,\dot{q},v,u) \\ &= \bar{B}^{-1}(q)[\bar{M}(q)u + \bar{V}_m(q,\dot{q})v + \bar{F}(v)] \end{aligned} \tag{11}$$

Substituting parameters of the mobile robot platform, Eq. (11) becomes [7]:

$$\begin{aligned} \tau &= \begin{bmatrix} \tau_{rw} \\ \tau_{lw} \end{bmatrix} = \begin{bmatrix} \frac{r}{2}m + \frac{1}{r}I_w & \frac{r}{2b}I + \frac{b}{r}I_w \\ \frac{r}{2}m + \frac{1}{r}I_w & -\frac{r}{2b}I - \frac{b}{r}I_w \end{bmatrix} u + \\ &\frac{r}{2b}m_c d\omega \begin{bmatrix} 1 & -b \\ -1 & -b \end{bmatrix} v \\ &m = m_c + 2m_w \\ &I = m_c d^2 + 2m_w b^2 + I_c + 2I_m \end{aligned} \tag{12}$$

where

τ_{rw} and τ_{lw} : the motors' torques which act on the right and the left wheels; m_c and m_w are the mass of body and wheel with a motor; d is the distance between the geometric and mass center of WMR; I_c is the inertia moments of the body about the vertical axis through WMR mass center; I_w, inertia moment of the a wheel and motor about the wheel axis; I_m, inertia moment the wheel with the motor about the wheel diameter.

It is possible to convert the dynamic control problem into the kinematics control problem.

$$\begin{aligned} \dot{q} &= S(q)v \\ \dot{v} &= u \end{aligned} \tag{13}$$

Eq. (13) represents a state-space description of the nonholonomic mobile robot. To make use of (13), it is assumed that all the dynamical quantity $\bar{M}(q)$, $\bar{F}(v)$, $\bar{V}(q,\dot{q})$ of the mobile robot are exactly known and disturbance $\bar{\tau}_d = 0$. Defining $x = \left[q^T \mathrm{v}^T\right]^T$, Eq. (13) can be rewritten as

$$\dot{x} = f(x) + g(x)u$$

C. Path Tracking Problem

Let a reference mobile robot be prescribed as

$$\begin{aligned} &\dot{x}_r = v_r \cos\varphi_r, \dot{y}_r = v_r \sin\varphi_r, \dot{\varphi}_r = \omega_r \\ &q_r = \left[x_r \quad y_r \quad \varphi_r\right]^T, v_r = \left[v_r \quad \omega_r\right]^T \end{aligned} \tag{14}$$

Find a smooth angular velocity control input $v_p(t) = f_p(e_p, v_r, K)$ such that $\lim_{t\to\infty}(q_r - q) = 0$, where e_p, v_r and K are the tracking errors, the reference velocity vector, and the controller gain vector, respectively. Then, compute the torque input $\tau(t)$ for (1), such that $v \to v_p$ as $t \to \infty$.

The point $P(x_p, y_p)$ can be derived from $Po(x, y)$:

$$\begin{cases} x_p = x + d\cos\phi \\ y_p = y + d\sin\phi \\ \phi_p = \phi \end{cases} \tag{15}$$

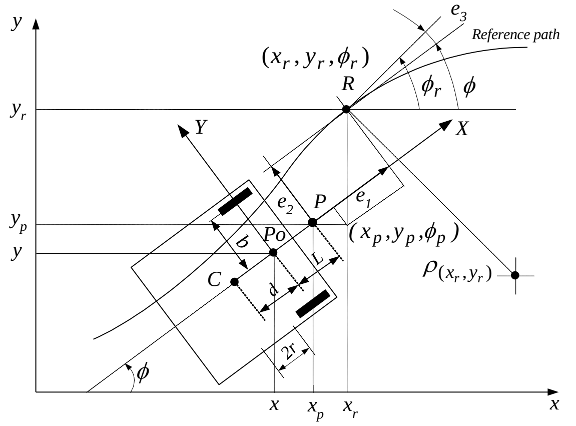

Figure 2. Scheme for deriving WMR kinematics equations

where d is the distance between P and Po. The derivative of (15) yields

$$\begin{cases} \dot{x}_p = \dot{x} - d\omega\sin\phi \\ \dot{y}_p = \dot{y} + d\omega\cos\phi \\ \dot{\phi}_p = \omega \end{cases} \tag{16}$$

It is assumed that a reference point $R(x_r, y_r)$ on the reference path is moving at the constant velocity of v_r with the orientation angle ϕ_r. The dynamic equation is shown below:

$$\begin{cases} \dot{x}_r = v_r\cos\phi_r \\ \dot{y}_r = v_r\sin\phi_r \\ \dot{\phi}_r = \omega_r \end{cases} \tag{17}$$

where φ_r is defined as the angle between v_r and x-axis and ω_r is the rate of change of v_r.

The relationship between v, ω and the angular velocities of two driving wheels is the following:

$$\begin{bmatrix} \omega_{rw} \\ \omega_{lw} \end{bmatrix} = \begin{bmatrix} 1/r & b/r \\ 1/r & -b/r \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{18}$$

where ω_{rw}, ω_{lw} represent the angular velocities of the right and the left wheels, b is distance from WMR's center point to the driving wheel and r is the radius of wheel.

III. PATH TRACKING CONTROLLER DESIGN

There are several approaches to select a velocity control input v for the system (7). In this section, the control v is converted into torque control τ for the actual physical mobile robot; that is to say, τ in (11) is selected so that (13) exhibits the desired behavior motivating the specific choice of the velocity v. This allows the steering system commands v to be converted to torques τ that take into account the mass, friction and others parameters of the actual mobile robot.

The scheme of errors measurement is shown in Fig. 2. The tracking errors $e_p = [e_1, e_2, e_3]^T$ are defined as following

$$e_p = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_p \\ y_r - y_p \\ \varphi_r - \varphi_p \end{bmatrix} \tag{19}$$

The first derivative of errors yields

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} -1 & e_2 \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix} . \begin{bmatrix} v_p \\ \omega_p \end{bmatrix} + \begin{bmatrix} v_r\cos e_3 \\ v_r\sin e_3 \\ \omega_r \end{bmatrix} \tag{20}$$

The Lyapunov function candidate is chosen as

$$V_o = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1-\cos e_3}{k_2} \geq 0 \tag{21}$$

The derivative of V_o becomes

$$\begin{aligned} \dot{V}_o &= e_1\dot{e}_1 + e_2\dot{e}_2 + \frac{\sin e_3}{k_2}\dot{e}_3 \\ &= e_1(-v_p + v_r\cos e_3) + \frac{\sin e_3}{k_2}(\omega_r - \omega_p + k_2 e_2 v_r) \end{aligned} \tag{22}$$

To achieve $\dot{V}_o \leq 0$, we choose

$$\begin{bmatrix} v_p \\ \omega_p \end{bmatrix} = \begin{bmatrix} v_r\cos e_3 + k_1 e_1 \\ \omega_r + k_2 v_r e_2 + k_3\sin e_3 \end{bmatrix} \tag{23}$$

where k_1, k_2 and k_3 are positive values.

To obtain the dynamic control (13), we have to find a nonlinear feedback control $u \in R^{n-m}$ so that $v \to v_p$ as $t \to \infty$.

First, we define an auxiliary velocity errors

$$e = v - v_p$$

$$\begin{bmatrix} e_4 \\ e_5 \end{bmatrix} = \begin{bmatrix} v_1 - v_{1p} \\ v_2 - v_{2p} \end{bmatrix} \tag{24}$$

where

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v \\ \omega \end{bmatrix} \text{ and } v_p = \begin{bmatrix} v_{1p} \\ v_{2p} \end{bmatrix} = \begin{bmatrix} v_p \\ \omega_p \end{bmatrix}$$

Define Lyapunov function V and its derivative as following:

$$V = \frac{1}{2}e^2 \geq 0$$

$$\dot{V} = e\dot{e}$$

To achieve $\dot{V} \leq 0$, we choose

$$\dot{e} = -K_4 e$$

$$\Rightarrow u = \dot{v}_p + K_4(v_p - v) \qquad (25)$$

Therefore, by making use of the acceleration control input (25), $e \to 0$, or $v \to v_p$, as $t \to \infty$.

where K_4 is a positive definite diagonal matrix given by $K_4 = k_4 I$.

III. SIMULATION RESULTS

To verify the effectiveness of the proposed controller, simulations have been done with the dynamics controller (23) and the kinematics controller (25) to track a defined reference smooth-curved path shown in Fig. 4. The controller parameters are $k_1 = 0.5$, $k_2 = 1000$, $k_3 = 0.05$ and $k_4 = 50$. The input disturbances are chosen to be 0. The WMR's parameters are known totally. The WMR's parameters and the initial values are given in TABLE 1. The speed of the mobile robot is designed at 100 mm/s.

The simulation results are given through Figs. 4-8.. The linear velocity of the center varies in the vicinity of 100mm/s in Fig. 6. The posture and the tracking point trajectory is shown in Fig. 9. The simulation results show that a good tracking performance can be achieved by the proposed controller.

In Fig. 4, it can be seen that the errors approach to zeros after 6 seconds; also, as the robot passes through the curved line there is a sudden change of ω_r which, in turn, make error e_3 about 17 degrees.

TABLE I. The WMR's parameters and initial values

Parameters	Values	Unit
(1) WMR's parameters		
b	0.105	m
d	0.08	m
R	0.025	m
m_c	10	kg
m_w	1	kg
I_w	3.75×10^{-4}	kgm^2
I_c	0.2081	kgm^2
I_m	4.96×10^{-4}	kgm^2
(2) Initial values		
x_r	0.28	m
x_p	0.27	m
y_r	0.4	m
y_p	0.39	m
ϕ_r	0	degree
ϕ_p	5	degree
v	0	mm/s
ω	0	rad/s

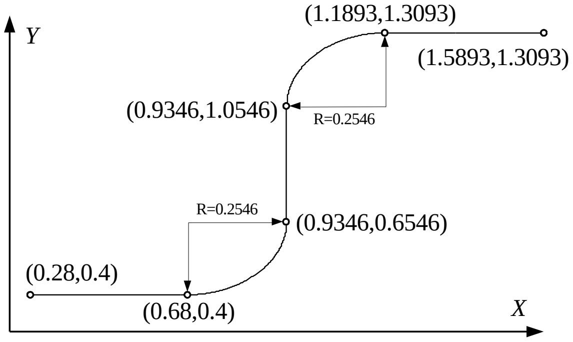

Figure 3. Reference trajectory

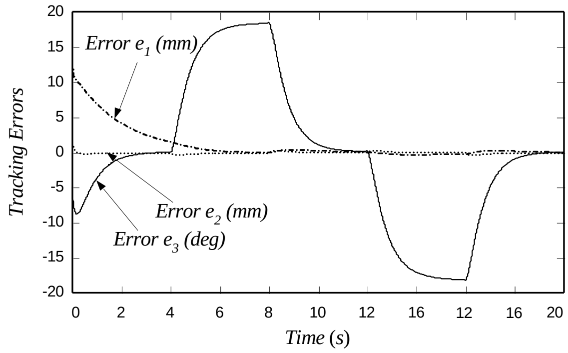

Figure 4. Tracking errors

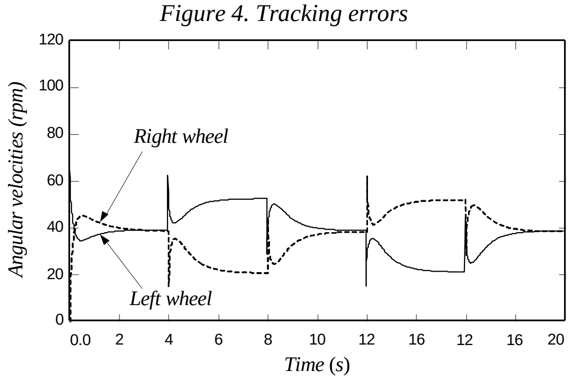

Figure 5. Angular velocities of two wheels

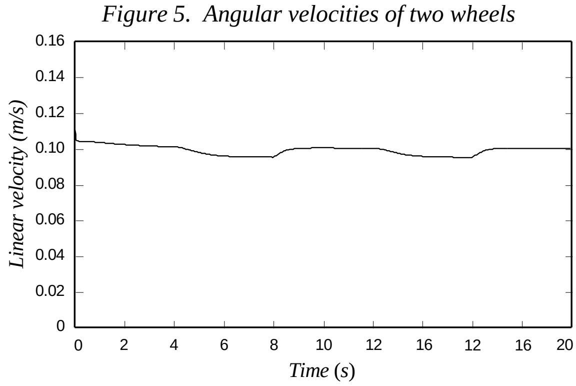

Figure. 6. Linear velocity of WMR's center

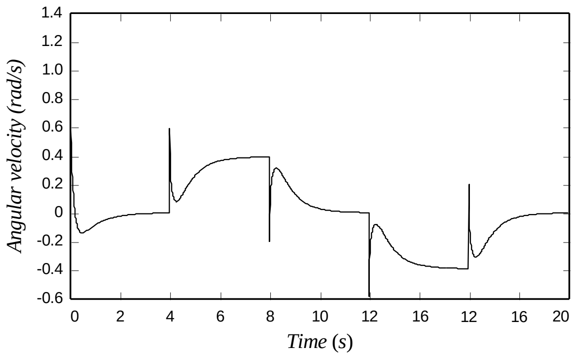

Figure 7. Angular velocity of WMR's center

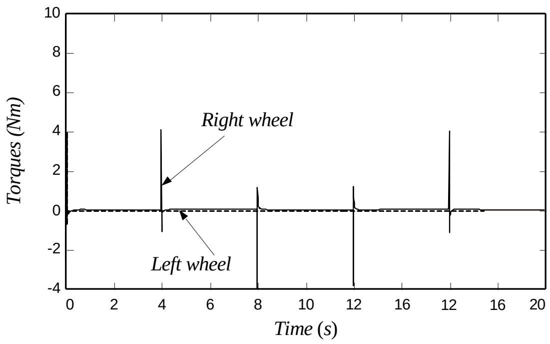

Fig. 8: Applied torques

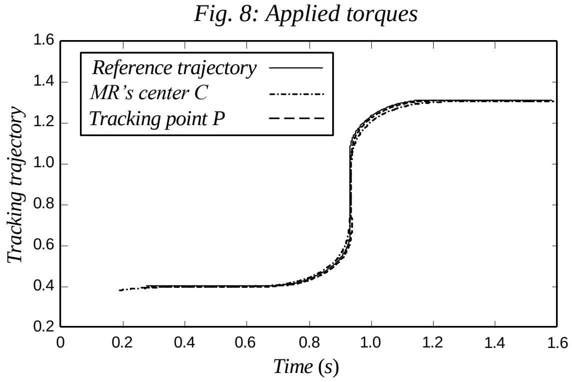

Fig. 9 WMR's movement tracking reference path

The mechanical model of the mobile robot is preliminary designed in the Fig. 13.

IV. CONTROL SYSTEM SCHEME

The control system scheme is based on the integration of a computer-based vision and an STM32-based motion controller. The image is processed using the computer and the mobile robot's wheels are drived via the motion controller with a servo BLDC motor driver (Figs.11-12). The configuration diagram of the total control system is shown in Fig. 10. For the operation, the image is processed to extract the feature of errors for the control law. The result of the processing, the torque command, is sent to the low level to control the mobile robot motion. The designed system is available, but the real system are not yet to finished at the time of this writing. The real mobile robot, control hardware and the experiments will be included in the future.

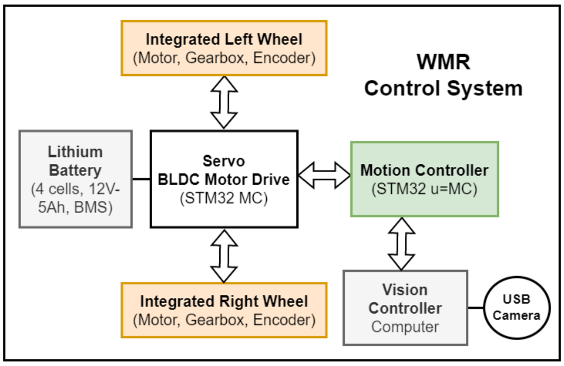

Figure 10. The configuration of the control system

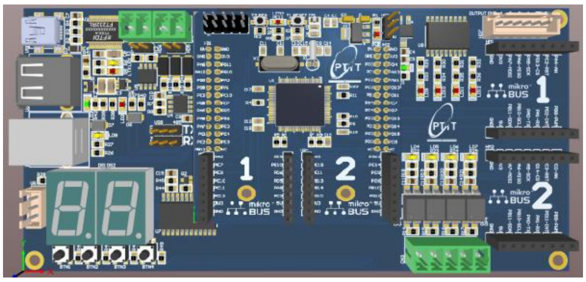

Figure 11. Motion controller is based on STM32

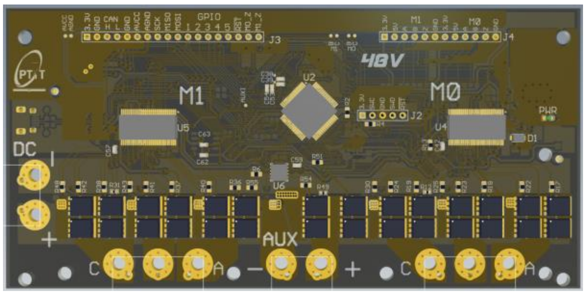

Figure 12. BLDC motor driver

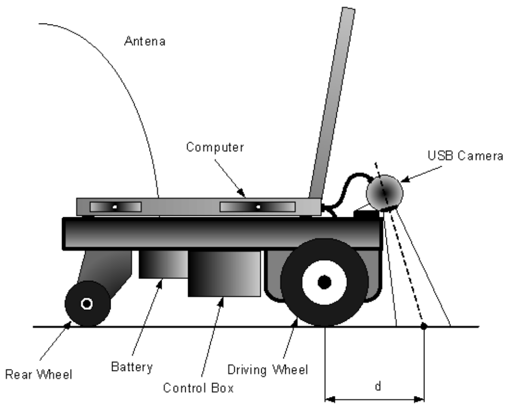

Figure 13. The model of the mobile robot

V. CONCLUSIONS

A simple nonlinear controller for a two-wheeled mobile robot based on backstepping design method has been introduced. The mobile robot is considered in terms of dynamics with exactly known parameters. The controller is stable in the sense of Lyapunov stability. Also, a simple way of measuring the errors for deriving the control law is proposed. The simulation results show that the controller is possible to implement in the future.

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MỘT BỘ ĐIỀU KHIỂN BÁM ĐƯỜNG ĐƠN GIẢN CHO ROBOT DI ĐỘNG HAI BÁNH XE

Tóm tắt: Trong bài báo này, một bộ điều khiển bám đường dựa trên việc kết hợp động học và mô-men hồi phục được ứng dụng vào một robot di động dạng hai bánh xe. Robot di động này được xem xét theo mô hình động lực học theo hệ tọa độ Cartesian và các tham số động lực học của nó được biết cụ thể chính xác. Để thiết kế bộ điều khiển, ta định nghĩa các sai số bám đường tham chiếu, sau đó thiết kế bộ điều khiển động học và bộ điều khiển mô-men để bảo đảm sai số hội tụ tiệm cận 0. Kết quả mô phỏng được thực hiện để chứng minh sự hiệu quả của bộ điều khiển.

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