

DOA ESTIMATION METHOD USING UNIFORM CIRCULAR ANTENNA ARRAY BASED ON LSTM

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Abstract—The Direction of Arrival plays an important role in wireless communication systems with many applications such as surveillance systems, radars, automatic collision avoidance, and detection systems. It is a critical signal processing problem, and various methods have been developed to optimize its accuracy. In this paper, the Long-short term memory network model is used to estimate the direction of incoming signals for the Uniform Circular Antenna array. The performance of this method is evaluated based on the RMSE parameter and compared with the MUSIC algorithm in different cases such as the deviation of incident angle of radiation sources, and signal-to-noise ratio...

Keywords — Direction of arrival estimation, Uniform Circular Antenna, Long-Short Term Memory, Machine Learning, Artificial Intelligence...

I. INTRODUCTION

With the strong development of technology in the 4.0 era, artificial intelligence (AI) also has a big development. They are commonly used in life such as devices that can recognize faces, and voices,... or medical devices which help doctors in the process of treating and diagnosing diseases... Deep learning (DL), which is a field of artificial intelligence, has been studied and applied in many fields including the DOA [1] [2] problem. For details, it is researched in improving the accuracy and speed of DOA estimation. In practice, the incoming sources are always random and not fixed, so the use of classical algorithm such as MUSIC [3] when assuming the number of incoming sources in advance can lose the generality of the problem and gives incorrect results. Besides, it is difficult to estimate DOA with high resolution. For deep learning techniques, a

sufficiently large dataset is put into 'training', so the computer can learn and estimate the DOA value with higher resolution and better accuracy. However, data collection and real data processing are often quite difficult, so in this paper, a simulation database is used with different incident angles to the UCA [4] [5] antenna array on the background of random noise. This database is normalized by the data preprocessor and trained by the LSTM (Long Short-Term Memory) network [6], so the computer can estimate the arrival direction. Moreover, the LSTM network is also combined with the Adam optimal function [7], which significantly improves the accuracy of the problem.

This paper is organized as follows. The received signal model and LSTM network are presented in Section 2. Section 3 analyzes the experiment results. The evaluation and conclusion are shown in Section 4.

II. MATERIALS AND METHODS

A. Received signal model.

This research utilizes the Uniform Circular Antenna Array (UCA) with M elements. Each antenna element works as an isotropic receiver. Figure 1 illustrates the signals to the antenna system, assuming the signal lies in the same azimuth plane as the antenna array.

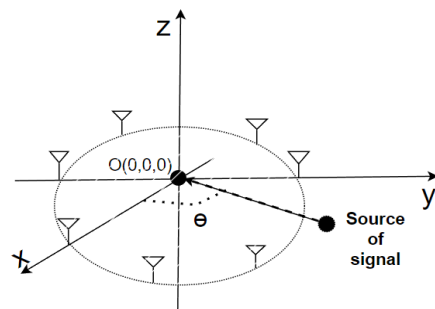


Figure 1. UCA antenna array

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Consider a problem with M equally spaced antenna elements in a circle of fixed radius R and the arc length between two adjacent antenna elements being d . The coordinate of the m^{th} antenna is determined by:

$$x_m = R \cos\left(\frac{2\pi m}{M}\right), \quad (1)$$

$$y_m = R \sin\left(\frac{2\pi m}{M}\right), \quad (2)$$

According to superposition principle, the signal received of each antenna is the sum of K signals, each has azimuth in turn of $\theta_1, \theta_2, \dots, \theta_k$. The signal at the m^{th} antenna element is:

$$\begin{aligned} y_m(t) &= \sum_{k=1}^K s_k(t) g_m e^{-j\frac{2\pi R}{\lambda} \cos(\theta_k - \frac{2\pi m}{M})} + \mathcal{E}(t) \\ &= \sum_{k=1}^K A_k(t) e^{-j\frac{2\pi R}{\lambda} \cos(\theta_k - \frac{2\pi m}{M})} + \mathcal{E}(t) \end{aligned} \quad (3)$$

where $A_k(t) = s_k(t) g_m$ with $s_k(t)$ is the complex amplitude of the k^{th} source, g_m is the gain factor of the antenna, $\mathcal{E}_m(t)$ is the noise received at the m^{th} antenna element of the array and $m = 1, 2, \dots, M$.

U is defined as a steering matrix of size $M \times K$, with the elements as follows:

$$u_{m,k} = e^{-j\frac{2\pi R}{\lambda} \cos(\theta_k - \frac{2\pi m}{M})} \quad (4)$$

then Eq.3 can be shortened as:

$$y(t) = UA(t) + \mathcal{E}(t), \quad (5)$$

where $y(t)$, $A(t)$ and $\mathcal{E}(t)$ is defined as below:

$$y(t) = [y_1(t), y_2(t), \dots, y_M(t)]^T$$

$$A(t) = [A_1(t), A_2(t), \dots, A_K(t)]^T \quad (6)$$

$$\mathcal{E}(t) = [\mathcal{E}_1(t), \mathcal{E}_2(t), \dots, \mathcal{E}_M(t)]^T$$

where $[\cdot]^T$ is a transposed matrix.

The signals received at the antenna array will be passed through a preprocessor. Therefore, the correlation matrix of size $M \times M$ that shows these received signals can be represented as bellow:

$$R_{xx} = E[y(t)y^H(t)] = USU^H + \mathcal{E}_n \quad (7)$$

where $E[\cdot]$ and $[\cdot]^H$ are the expectation and the Hermitian transpose, respectively. S and \mathcal{E}_n are correlation matrices of size $K \times K$ for signal and noise as follows:

$$S = E[s(t)s^H(t)] \quad (8)$$

$$\mathcal{E}_n = E[n(t)n^H(t)] \quad (9)$$

Moreover, Eq.7 can be rewritten as:

$$R_{xx} = USU^H + \sigma_{noise}^2 I \quad (10)$$

where I is an identity matrix of size $M \times M$, σ_{noise}^2 is the noise power. The correlation matrix is also called Hermitian matrix which is used as an input for the DOA estimation models.

B. LSTM

1. LSTM networks (Long Short-Term Memory networks)

LSTM (Long Short-Term Memory) network is an improved form of RNN (Recurrent neural network) [8], it can solve the problem of remembering long steps of RNN. The LSTM network has a carousel-like cell state. The structure of an LSTM "cell" is depicted in Figure 2.

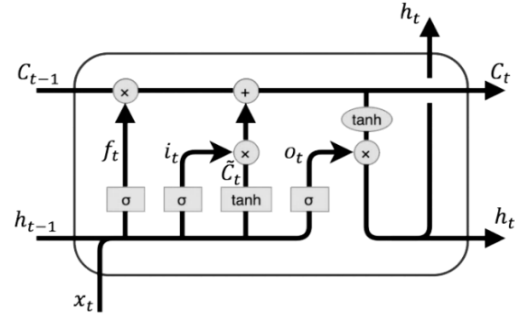


Figure 2. Structure of the LSTM module [8]

Specifically, in the t^{th} state of the LSTM model [8], the classic LSTM module structure includes inputs: c_{t-1} ; h_{t-1} – are the outputs in the $(t-1)^{\text{th}}$ state; and x_t , where x_t is the input in the t^{th} state of the model. Output includes c_t, h_t , where c_t is called cell state, h_t is hidden state at the t^{th} state.

$$\text{Forget gate: } f_t = \sigma(W_f * x_t + W_f * h_{t-1} + b_f) \quad (11)$$

$$\text{Input gate: } i_t = \sigma(W_i * x_t + W_i * h_{t-1} + b_i) \quad (12)$$

$$\text{Output gate: } o_t = \sigma(W_o * x_t + W_o * h_{t-1} + b_o) \quad (13)$$

$$\tilde{c}_t = \tanh(W_c * x_t + W_c * h_{t-1} + b_c) \quad (14)$$

$$c_t = f_t * c_{t-1} + i_t * \tilde{c}_t + b_c \quad (15)$$

$$h_t = o_t * \tanh(c_t) \quad (16)$$

where $0 < f_t, i_t, o_t < 1$; b_f, b_i, b_o are bias coefficients; W is the weight matrix.

There are many LSTM network models in different problems. In this paper, the LSTM network we use is shown in Figure 3. Three layers of LSTM are used and nodes are adjusted accordingly, thus helping to improve accuracy compared to RNN models.

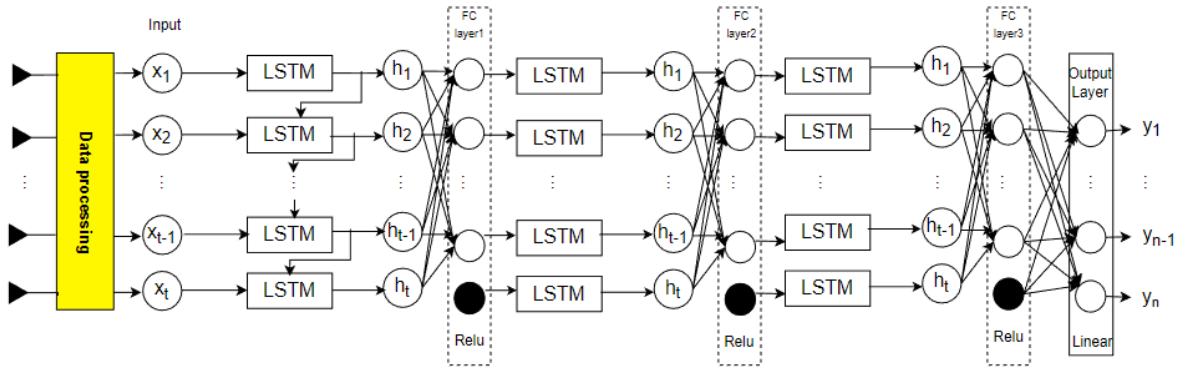


Figure 3. LSTM model for DOA

2. Data preprocessing

To ensure the accuracy of the DOA problem, the amount of training data will be quite large, so they will be processed through the data preprocessor to reduce bias. The output of the data preprocessor is the Hermitian matrix R_{xx} of size $M \times M$ as described in equation (7). However, since the input of the LSTM network must be real numbers, the matrix R_{xx} will be transformed into a matrix A [1] of size $2M \times M$ which consists of the real and imaginary parts of each element in the matrix R_{xx} :

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,M} \\ a_{2,1} & a_{2,2} & \dots & a_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ a_{2M,1} & a_{2M,2} & \dots & a_{2M,M} \end{bmatrix} \quad (17)$$

where $a_{i,j}$ with $i \leq M$ is the real part and $a_{i,j}$ with $M < i \leq 2M$ is the imaginary part.

According to the SVD method [9], the matrix A will be analyzed as follows:

$$A_{2M \times M} = W_{2M \times 2M} Y_{2M \times M} (V_{M \times M})^T \quad (18)$$

where W, V are orthogonal matrices, Y is a non-square diagonal matrix whose diagonal elements are $y_1 \geq y_2 \geq \dots \geq y_r \geq 0 = 0 = \dots = 0$ and r is the rank of matrix A.

Especially, based on the Truncated SVD method [9] which is a form of SVD, the matrix A can be approximated according to equation (19) by the sum of t matrices of rank = 1 (with $t < r$) as follows:

$$\begin{aligned} A \approx A_t &= W_t Y_t (V_t)^T \\ &= w_1 y_1 v_1^T + w_2 y_2 v_2^T + \dots + w_t y_t v_t^T \end{aligned} \quad (19)$$

And then, the error is defined with the Frobenius norm of the matrix as follows:

$$\|A - A_t\|_F^2 = \sum_{i=t+1}^r y_i^2 \quad (20)$$

Substituting $t = 0$, we have:

$$\|A\|_F^2 = \sum_{j=1}^r y_j^2 \quad (21)$$

Then, the percentage of information retained by the matrix is approximated through the formula:

$$P = \frac{\sum_{i=1}^t y_i^2}{\sum_{j=1}^r y_j^2} \quad (22)$$

So, if we want to remain about 90% information of the original matrix A, we need to find the minimum integer t that meets the condition as below:

$$\frac{\sum_{i=1}^t y_i^2}{\sum_{j=1}^r y_j^2} \geq 0.9 \quad (23)$$

For this problem, to retain 90% of the information on matrix A, t is equal to 2. Then the matrix $A \approx A_t$ which has size $2M \times 2$ as follows:

$$A_t = \begin{bmatrix} \bar{a}_{1,1} & \bar{a}_{1,2} \\ \vdots & \vdots \\ \bar{a}_{2M,1} & \bar{a}_{2M,2} \end{bmatrix} \quad (24)$$

Since $M > t$, $M=3,4,5\dots$ Therefore, this is true for all real UCA antenna systems. The matrix A_t is then transformed into vector $X = [\bar{a}_{1,1}, \dots, \bar{a}_{2M,2}]^T$ (25) as input to the LSTM network where $[\cdot]^T$ is the transpose matrix.

3. Data Labeling

The input is defined as vector $X(\theta_1, \theta_2 \dots \theta_K)$, with K is the number of arrival sources, corresponding to the incoming signal sources at angle $\theta_1, \theta_2 \dots \theta_K$. Vector X here is described by Eq.25

In this research, a labeling method called one hot encoding with multiple labels is used to label the data. $[y(\theta_1, \theta_2 \dots \theta_K)]_{label}$ is the corresponding label of the incoming signals. With 241 outputs corresponding to the incoming angles in the range $[-120^\circ \div 120^\circ]$ with a jump of 1° , a formula is defined as follows:

$$[y(\theta_1, \dots, \theta_K)]_{label} = \begin{cases} 1, & \text{at } \theta_i \text{ with } i = 1, \dots, K \\ 0, & \text{otherwise} \end{cases} \quad (26)$$

For example, with two incoming signals, the label form will be [1 0 1 ... 0 0].

Therefore, the output of the LSTM network corresponding to the input $X(\theta_1, \theta_2 \dots \theta_K)$ is $y(\theta_1, \theta_2 \dots \theta_K)$.

4. MUSIC algorithm

MUSIC is a classical algorithm for estimating DOA. For this method, the signal space is estimated using the eigenvalues of the covariance matrix. The algorithm consists of four main steps: find the covariance matrix, calculate eigenvalues and eigenvectors, signal division and calculate the spatial spectrum to get the DOA.

5. Evaluation parameters

To evaluate the accuracy of two methods, the LSTM model and the MUSIC algorithm, two error functions namely MSE and RMSE are researched and used.

a. MSE

To evaluate the model during training, the network utilizes the mean square error function (MSE) which is defined as:

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (27)$$

b. RMSE

This study uses the root mean square error function (RMSE) to evaluate the performance of the model and algorithm:

$$RMSE = \sqrt{\frac{1}{NK} \sum_{k=1}^K \sum_{n=1}^N (\theta_k - \hat{\theta}_{k,n})^2} \quad (28)$$

where K is the number of incident sources, N is the number of trials, θ_k and $\hat{\theta}_{k,n}$ in turn are the incident angle of the k^{th} source and the estimated angle of the k^{th} source at the n^{th} trial.

III. SIMULATION RESULTS

A. Simulation parameters

During the simulation, a 10-element UCA antenna array with $d = \frac{\lambda}{2}$, $R = \frac{5\lambda}{2\pi}$ and frequency $f = 2\text{GHz}$ is used. Received signals should be a narrowband and uncorrelated, simulated signal generated according to equation (3) with signal-to-noise ratio $\text{SNR} = 10\text{dB}$, snapshot = 400, number of incoming sources is $K = 2$ and the number of trials is $N = 1000$ to calculate the value of the RMSE parameter.

The LSTM network used in this paper is shown in Figure 3. The LSTM network is designed with 1 input

layer, 3 LSTM layers, 3 fully-connected layers using the Relu activation function, and the last output layer using the linear activation function. The sizes of those layers are shown in Table 1. Assuming that each incoming signal is in the range $[-120^\circ, 120^\circ]$ and the angular resolution is 1° , So the number of nodes of the output layer will be 241. The spatial spectrum is also calculated with an angular resolution of 1° so, we have a total of 241 grids with $\theta_1 = -120, \theta_2 = -119, \dots, \theta_{241} = 120$.

Table 1. Number of nodes in each layer

Layer name	Number of nodes
LSTM1	512
FC1	180
LSTM2	512
FC2	400
LSTM3	512
FC3	256
Output	241

In this paper, it is assumed that there are 2 incoming signal sources and the angular difference between the incoming sources is taken as $\Delta_\theta = \{2^\circ, 4^\circ, \dots, 20^\circ\}$. The DOA of the first signal θ is generated by sampling in the range $[-120^\circ, 120^\circ - \Delta]$, with a sampling step of 1° then the DOA of the second signal will be $\theta + \Delta$. The jump between pairs of angles is explained as if the first pair of angles is $(-120^\circ, -118^\circ)$ then the next pair will be $(-119^\circ, -117^\circ)$. Therefore, if the first incoming source DOA is 20° with $\Delta_\theta = 10^\circ$ it is easy to calculate that the 2nd source DOA is 30° . These samples then is used to compute the X vectors as input to the LSTM network according to equation (25) with the corresponding labels as equation (26) SNR of incoming signals is 10dB. For the training process, the learning rate is 0.001, the batch size is 1024 and the number of epochs is 100. During the training process, the MSE loss function is used to continuously update the parameters in the network to the training results. In addition, the network also uses the Adam optimization algorithm to increase accuracy as well as optimize the training time.

B. Simulation results

This section presents the results of LSTM method in various scenarios and then compares them with the results of MUSIC method.

First, assuming that $\text{SNR} = 10\text{dB}$ and the signal sources are uncorrelated. When the difference between 2 incident angles is 10° , $\text{RMSE} \approx 0.02$ for LSTM method while for MUSIC method, $\text{RMSE} \approx 0.35$. However, when the 2 incident angles are very close with $\Delta_\theta = 2^\circ$, the LSTM method performs quite well with $\text{RMSE} \approx 0.05$. In contrast, the MUSIC method cannot give an accurate outcome because it can estimate only 1 incidence angle. Additional details are shown in Table 2 and Table 3.

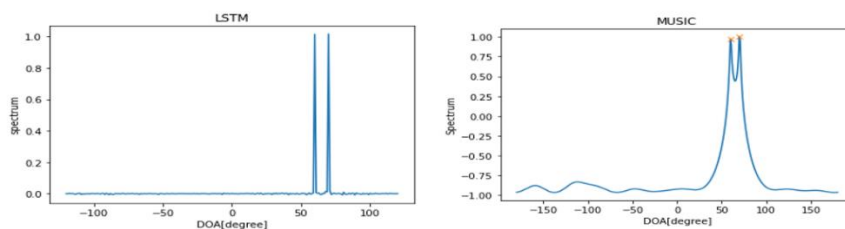


Figure 4. The signal spectrum of incident source with angular distance of 100 based on LSTM and MUSIC methods

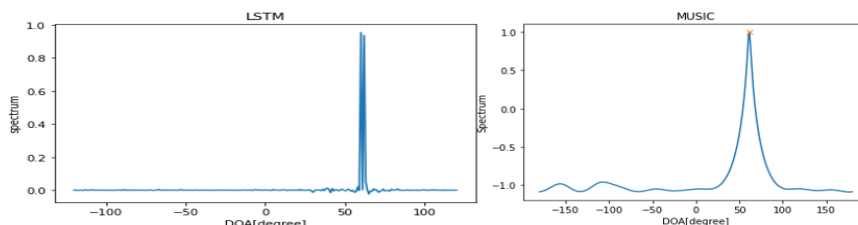


Figure 5. The signal spectrum of incident source with angular distance of 20 based on LSTM and MUSIC methods.

Table 2. The angles resolution of LSTM (SNR = 10dB)

Δ_θ	Input(degree)		Output(degree)		RMSE
2°	60	62	59.99	62.05	0.05
4°	60	64	60	64.02	0.03
6°	60	66	60	65.99	0.03
8°	60	68	60	68.02	0.02
10°	60	70	60	69.99	0.02

accurately though the difference between them is very small.

Next, the influence of a signal-to-noise ratio (SNR) on the accuracy of both LSTM and MUSIC methods is evaluated when the difference between 2 incident sources is 10°. SNR is taken in 3 cases -3dB, 0dB, and 5dB with 50 samples. When SNR increases, LSTM gives more accurate results. At SNR = -3dB, values of the estimated angle pair and actual angle pair are distinct. At SNR = 0dB the difference has significantly reduced and at SNR = 5dB, these 2 values are approximate. The results are shown in Figure 6, 7 and 8.

Table 3. The angles resolution of MUSIC (SNR = 10dB)

Δ_θ	Input(degree)		Output (degree)		Result
2°	60	62	60.66	X	False
4°	60	64	X	63.67	False
6°	60	66	60.66	65.68	True
8°	60	68	59.66	67.68	True
10°	60	70	59.66	69.69	True

From these result tables above, the LSTM method provides better simulation results than the MUSIC algorithms. The LSTM estimates incident angles more

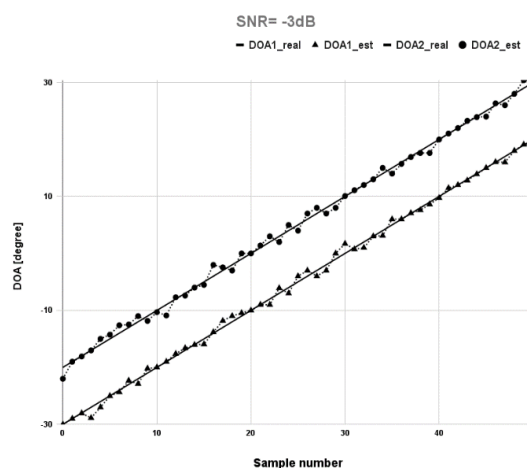


Figure 6. The actual and estimated DOA values of the two signals of the LSTM method with SNR = -3 dB

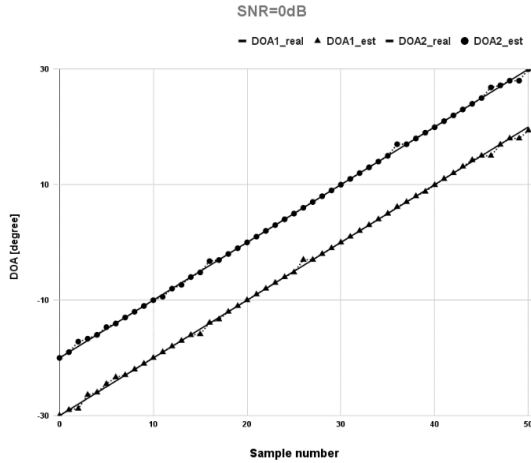


Figure 7. The actual and estimated DOA values of the two signals of the LSTM method with SNR=0 dB

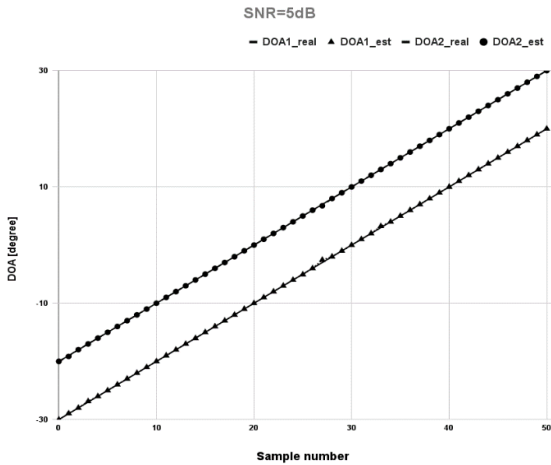


Figure 8. The actual and estimated DOA values of the two signals of the LSTM method with SNR=5 dB



Figure 9. Compare RMSE (degrees) of DOA algorithm based on LSTM and MUSIC methods with different SNR values

For further evaluation, an investigation is carried out with pairs of incident angles at 60° and 70° , and SNR values in range of $[0dB \div 10dB]$ with $1dB$ step. The results are expressed in Figure 9.

From all the results above, the LSTM method shows better results than MUSIC. Especially, $RMSE \approx 0$ when $SNR > 8dB$.

Finally, an examination of the influence of snapshot quantities on the accuracy of LSTM and MUSIC algorithms is performed by using the angles pair at 60° and 70° , and the snapshot in range of $[10 \div 100]$ with step at 10. The simulation results are represented in Figure 10.

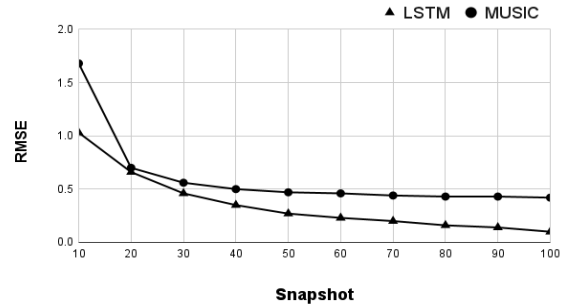


Figure 10. Compare RMSE (degrees) of LSTM and MUSIC at an angle ($60^\circ, 70^\circ$) with different snapshots.

The above results demonstrate that the more snapshots, the more accurate both methods become. However, the LSTM method brings better outcomes with minor RMSE errors when the snapshot values are from 10 to 100.

IV. CONCLUSION

This research uses an LSTM model to estimate the incident wave direction for the UCA antenna array. The model uses Adam optimization algorithm to increase the accuracy and the MSE error function to adjust antenna network parameters for the most accurate results. The LSTM's performance is compared to the MUSIC algorithm's using RMSE parameter, showing that the LSTM network's results are better. However, despite the high accuracy, the LSTM network is still quite complex, so it needs to be optimized to reduce the computational volume but still ensure accuracy. The LSTM network also needs improvement to operate well under lower SNR conditions or when there are more than two incoming signal or the incoming signal are correlated.

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PHƯƠNG PHÁP ƯỚC TÍNH HƯỚNG SÓNG TỚI SỬ DỤNG DÀN ANTEN UCA DỰA TRÊN KỸ THUẬT HỌC SÂU LSTM

Tóm tắt: Ước lượng hướng sóng tới (DOA) có vai trò quan trọng trong thời đại công nghệ phát triển hiện nay với các ứng dụng như : hệ thống giám sát, radar, các hệ thống tự động phát hiện và tránh xung đột, hệ anten thông minh... Đây là bài toán rất quan trọng và đã được phát triển bằng nhiều phương pháp khác nhau để tối ưu độ chính xác. Trong nghiên cứu này, chúng tôi sử dụng mô hình mạng LSTM (Long-short term memory) để ước lượng hướng sóng tới cho mảng anten tròn (UCA). Hiệu suất của phương pháp được đánh giá qua tham số RMSE và so sánh với thuật toán MUSIC trong các trường hợp khác nhau như: độ lệch góc tới của các nguồn bức xạ, tỉ số tín hiệu trên nhiễu (SNR)...



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